Support Theory for Preconditioning -- Rectangular Factorizations

Researchers at Sandia have developed a new methodology for analyzing the effectiveness of preconditioners for linear systems, called *support theory*. Preconditioners are critical to get good performance from iterative solvers, but unfortunately, the generation of good preconditioners often involves as much art as science. Support theory provides a new way of thinking about preconditioners and provides useful analysis tools which can bound eigenvalues and condition numbers. This way one can gain new insights into existing preconditioners and develop new preconditioners with provably good properties.

One part of this project has focused on rectangular factorizations of matrices. While the traditional LU factorization of a square matrix factors a matrix into A=LU, where L and U are both square and also upper/lower triangular, one can also think of rectangular factors. Specifically, for many symmetric matrices one can quickly factor $A=VV^T$, where V is sparse but has more columns than rows. Although not useful in a direct method, one can construct a preconditioner as $A' = V'V'^T$, where V' is a subset of the columns in V. Spanning tree preconditioners (described in an earlier news note) fit into this framework. Support theory can be used to analyze such preconditioners.

Currently, we are investigating whether rectangular factorizations can lead to good approximate inverse preconditioners. The main idea is to approximate an inverse as $A^{-1} \approx YY^T$, where Y is sparse and rectangular. This is different from all existing approximate inverse schemes. Approximate inverse preconditioners have the advantage that one only needs to perform matrix-vector multiplications to apply them, which is usually fast on parallel computers. Several rectangular factored approximate inverse algorithms are currently under investigation.

For more information, please see www.cs.sandia.gov/~bahendr/support.html or contact Erik Boman (egboman@sandia.gov) or Bruce Hendrickson (bah@cs.sandia.gov).
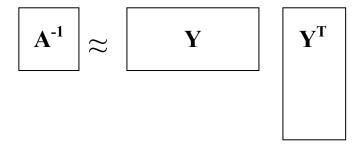

Figure: Approximate inverse using rectangular factors.